Rasterization (scan conversion)

- Final step in pipeline: rasterization
- From screen coordinates (float) to pixels (int)
- Writing pixels into frame buffer
- Separate buffers:
  - depth (z-buffer),
  - display (frame buffer),
  - shadows (stencil buffer),
  - blending (accumulation buffer)

Rasterizing a line

Digital Differential Analyzer (DDA)

- Represent line as
  \[ y = mx + h \]
  where \[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \]

- Then, if \( \Delta x = 1 \) pixel, we have \( \Delta y = m \Delta x = m \)

Digital Differential Analyzer

- Assume \( \text{write}_\text{pixel}(\text{int } x, \text{ int } y, \text{ int } value) \)
  for \( (i = x_1; i <= x_2; i++) \)
  \{
    y += m;
    \text{write}_\text{pixel}(i, \text{ round}(y), \text{ color});
  \}
- Problems:
  - Requires floating point addition
  - Missing pixels with steep slopes: slope restriction needed

Digital Differential Analyzer (DDA)

- Assume \( 0 \leq m \leq 1 \)
- Exploit symmetry
- Distinguish special cases
  But still requires floating point additions!
Bresenham's Algorithm I

- Eliminate floating point addition from DDA
- Assume again $0 \leq m \leq 1$
- Assume pixel centers halfway between integers

Bresenham's Algorithm II

- Decision variable $a - b$
  - If $a - b > 0$ choose lower pixel
  - If $a - b \leq 0$ choose higher pixel
- Goal: avoid explicit computation of $a - b$
- Step 1: re-scale $d = (x_2 - x_1)(a - b) = \Delta x(a - b)$
  - $d$ is always integer

Bresenham's Algorithm III

- Compute $d$ at step $k+1$ from $d$ at step $k$
- Case: $j$ did not change ($d_k > 0$)
  - $a$ decreases by $m$, $b$ increases by $m$
  - $(a - b)$ decreases by $2m = 2(\Delta y / \Delta x)$
  - $\Delta x(a - b)$ decreases by $2\Delta y$

Bresenham's Algorithm IV

- Case: $j$ did change ($d_k \leq 0$)
  - $a$ decreases by $m - 1$, $b$ increases by $m - 1$
  - $(a - b)$ decreases by $2m - 2 = 2(\Delta y / \Delta x - 1)$
  - $\Delta x(a-b)$ decreases by $2(\Delta y - \Delta x)$

Bresenham's Algorithm V

- So $d_{k+1} = d_k - 2\Delta y$ if $d_k > 0$
- And $d_{k+1} = d_k - 2(\Delta y - \Delta x)$ if $d_k \leq 0$
- Final (efficient) implementation:

```c
void draw_line(int x1, int y1, int x2, int y2) {
    int x = y = y0;
    int twice_dx = 2 * (x2 - x1), twice_dy = 2 * (y2 - y1);
    int twice_dy_minus_twice_dx = twice_dy - twice_dx;
    int d = twice_dx / 2 - twice_dy;
    for (x = x1; x <= x2; x++) {
        write_pixel(x, y, color);
        if (d > 0) d -= twice_dy;  
        else y++, d = twice_dy_minus_twice_dx;
    }
}
```

Bresenham's Algorithm VI

- Need different cases to handle $m > 1$
- Highly efficient
- Easy to implement in hardware and software
- Widely used
Outline

- Scan Conversion for Lines
- Scan Conversion for Polygons
- Antialiasing

Scan Conversion of Polygons

- Multiple tasks:
  - Filling polygon (inside/outside)
  - Pixel shading (color interpolation)
  - Blending (accumulation, not just writing)
  - Depth values (z-buffer hidden-surface removal)
  - Texture coordinate interpolation (texture mapping)
- Hardware efficiency is critical
- Many algorithms for filling (inside/outside)
- Much fewer that handle all tasks well

Filling Convex Polygons

- Find top and bottom vertices
- List edges along left and right sides
- For each scan line from bottom to top
  - Find left and right endpoints of span, xl and xr
  - Fill pixels between xl and xr
  - Can use Bresenham’s algorithm to update xl and xr

Concave Polygons: Odd-Even Test

- Approach 1: odd-even test
- For each scan line
  - Find all scan line/polygon intersections
  - Sort them left to right
  - Fill the interior spans between intersections
- Parity rule: inside after an odd number of crossings

Edge vs Scan Line Intersections

- Brute force: calculate intersections explicitly
- Incremental method (Bresenham’s algorithm)
- Caching intersection information
  - Edge table with edges sorted by y_{min}
  - Active edges, sorted by x-intersection, left to right
- Process image from smallest y_{min} up

Concave Polygons: Tessellation

- Approach 2: divide non-convex, non-flat, or non-simple polygons into triangles
- OpenGL specification
  - Need accept only simple, flat, convex polygons
  - Tessellate explicitly with tessellator objects
  - Implicitly if you are lucky
- Most modern GPUs scan-convert only triangles
Flood Fill

- Draw outline of polygon
- Pick color seed
- Color surrounding pixels and recurse
- Must be able to test boundary and duplication
- More appropriate for drawing than rendering

Outline

- Scan Conversion for Lines
- Scan Conversion for Polygons
- Antialiasing

Aliasing

- Artifacts created during scan conversion
- Inevitable (going from continuous to discrete)
- Antialiasing (name from digital signal processing): we sample a continues image at grid points
- Effect
  - Jagged edges
  - Moire patterns

Moire pattern from sandlotscience.com

More Aliasing

Antialiasing for Line Segments

- Use area averaging at boundary
- (a) is aliased; (b) is antialiased
- (c) is aliased + magnified
- (d) is antialiased + magnified

Antialiasing by Supersampling

- Mostly for off-line rendering (e.g., ray tracing)
- Render, say, 3x3 grid of mini-pixels
- Average results using a filter
- Can be done adaptively
  - Stop if colors are similar
  - Subdivide at discontinuities
Supersampling Example

• Other improvements
  – Stochastic sampling: avoid sample position repetitions
  – Stratified sampling (jittering): perturb a regular grid of samples

Temporal Aliasing

• Sampling rate is frame rate (30 Hz for video)
• Example: spokes of wagon wheel in movies
• Solution: supersample in time and average
  – Fast-moving objects are blurred
  – Happens automatically with real hardware (photo and video cameras)
    • Exposure time is important (shutter speed)
  – Effect is called motion blur

Wagon Wheel Effect

Source: YouTube

Motion Blur Example

Achieve by stochastic sampling in time

T. Porter, Pixar, 1984
16 samples / pixel / timestep

Depth of Field

Wide depth of field Narrow depth of field
digital-photography-school.com

Summary

• Scan Conversion for Polygons
  – Basic scan line algorithm
  – Convex vs concave
  – Odd-even rules, tessellation

• Antialiasing (spatial and temporal)
  – Area averaging
  – Supersampling
  – Stochastic sampling